SCHEDULING PROJECTS WITH REPEATING ACTIVITIES

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ABSTRACT: Construction contractors are often faced with projects containing multiple units wherein activities repeat from unit to unit. These projects require schedules that ensure the uninterrupted usage of resources from an activity in one unit to a similar activity in the next unit. The critical path method (CPM) cannot assure this requirement because only technical precedence and resource availability constraints are explicitly shown in CPM networks. The repetitive scheduling method (RSM) described in this paper recognizes the technical constraints of CPM and also includes an additional resource continuity constraint to ensure continuous resource usage. RSM is a scheduling methodology that simplifies and generalizes various multiunit scheduling procedures previously proposed by several authors and known by a number of different names. It applies to both vertical and horizontal projects containing either discrete or continuous activities. An RSM schedule is presented graphically as an *X-Y* plot of a series of production lines, each of which represent a repetitive activity. RSM introduces the control point as a new concept for positioning successive production lines that may either diverge or converge depending upon their relative slopes. The control point between two successive production lines is located toward the first unit in the sequence of units if the lines diverge, and toward the last unit in the sequence if the lines converge. RSM also introduces the controlling sequence of activities as a new concept for the determination of the project duration. This sequence includes activities between control points on successive unit production lines and extends from project start to project finish. The controlling sequence may include both critical and noncritical activities.

INTRODUCTION

Construction contractors often encounter projects that contain several identical or similar units, such as floors in multistory buildings, houses in housing developments, meters in pipelines, or stations in highways. These multiunit projects are characterized by repeating activities, which in most instances arise from the subdivision of a generalized activity into specific activities associated with particular units. For example, a "Paint Walls" activity for a multistory building may be broken into "Paint First Floor Walls," "Paint Second Floor Walls," etc., where each floor is a significant unit of the overall project.

Activities that repeat from unit to unit create a very important need for a construction schedule that facilitates the uninterrupted flow of resources (i.e., work crews) from one unit to the next, because it is often this requirement that establishes activity starting times and determines the overall project duration. Hence, uninterrupted resource utilization becomes an extremely important issue.

The scheduling problem posed by multiunit projects with repeating activities is akin to the minimization of the project duration subject to resource continuity constraints as well as technical precedence constraints. The uninterrupted deployment of resources is not a problem addressed by the critical path method (CPM), nor by its resource-oriented extensions, such as time-cost trade-off, limited resource allocation, and resource leveling.

However, this need for the uninterrupted utilization of resources from an activity in one unit to the same (repeating) activity in the next unit is explicitly recognized by several scheduling methodologies that have been available for many years and have been called by a number of different names. For projects with discrete units, such as floors, houses, apartments, stores, or offices, names that have been used include: "Line of Balance" (O'Brien 1969; Carr and Meyer 1974; Halpin and Woodhead 1976; Harris and Evans 1977); "Construction Planning Technique" (Peer 1974; Selinger 1980); "Vertical Production Method" (O'Brien 1975; Barrie and Paulson 1978); "Time-Location Matrix Model" (Birrell 1980); "Time Space Scheduling Method" (Stradal and Cacha 1982); "Disturbance Scheduling" (Whiteman and Irwig 1988); or "Horizontal and Vertical Logic Scheduling for Multistory Projects" (Thabet and Beliveau 1994).

For highways, pipelines, tunnels, etc., where progress is measured in terms of horizontal length, the names used have included: "Time Versus Distance Diagrams" (Gorman 1972); "Linear Balance Charts" (Barrie and Paulson 1978); "Velocity Diagrams" (Dressler 1980); or "Linear Scheduling Method" (Johnston 1981; Chrzanowski and Johnston 1986; Russell and Casselton 1988).

Although each of these methods was developed to meet its own particular objectives, all of them are essentially alike in that they schedule the work in the project by plotting the progress of repeating activities against time. It is not difficult to argue that in the publications cited in the preceding text the presentations of these multiunit scheduling methods may have been more complicated than perhaps they should have been, and that fragmentation may have led to their limited acceptance by industry practitioners.

The objective of this paper is to integrate these methods into one generalized and simplified model, the repetitive scheduling method (RSM), that ensures continuous resource utilization and is applicable to both vertical and horizontal construction. Two new concepts that emerge from the development of RSM are control points and the controlling sequence. Therefore, a secondary objective of this work is to demonstrate the development and application of these new ideas.

RSM is not a complicated technique. It is a simple and easily applied scheduling methodology that follows naturally from the concepts and relationships found in CPM precedence networks. Hopefully, this presentation of RSM will be easily understood and accepted by construction planners as an additional and useful tool to schedule multiunit construction projects.

CPM MULTIUNIT SCHEDULING

Multiunit projects can be scheduled using commonly accepted CPM techniques, but continuous utilization of resources across repeating units cannot be assured when these CPM networks are

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Note. Discussion open until January 1, 1999. To extend the closing date one month, a written request must be filed with the ASCE Manager of Journals. The manuscript for this paper was submitted for review and possible publication on December 16, 1996. This paper is part of the ***Journal of Construction Engineering and Management***, Vol. 124, No. 4, July/August, 1998. ©ASCE, ISSN 0733-9634/98/0004-0269-0278/$8.00 + $.50 per page. Paper No. 14780.

used. This shortcoming is best illustrated by an example.

Fig. 1 is a CPM network prepared for a project consisting of three repeating units of work. The solid lines linking the activities within each unit and linking similar activities from unit to unit represent the technical precedence constraints in the network; for example, Activities *B*1, *C*1, and *A*2 cannot be started until Activity *A*1 is completed. The dashed lines linking similar activities from unit to unit represent resource availability constraints; for example, Activity *A*2 cannot begin until the crew of carpenters from Activity *A*1 is available.

Note that Units 1 and 3 each have five activities, *A*–*E*, but Unit 2 does not contain a *B* activity. Unit 2 also differs in that the individual activity durations are not the same as in Units 1 and 3. These differences reflect the various amounts of work needed to complete the activities of the unit.

The solution of the network in Fig. 1 results in a project duration of 18 days and a critical path that includes Activities *A*1, *C*1, *C*2, *D*2, *D*3, and *E*3. Typically, each unit in a repetitive network contains the same activities having the same durations, and the critical path passes through the network of activities in the first unit until an activity with a long duration is found. The path then passes through similar activities in successive units until the last unit in the sequence is reached, and continues through the last unit network until the final activity is completed. If all three units in Fig. 1 had been alike, the path would have included Activities *A*1, *C*1, *C*2, *C*3, *D*3, and *E*3. The shift in the path to include Activity *D*2 and not *C*3, as might be expected, is caused by the activity differences in Unit 2.

The links in this CPM network ensure that both technical precedence and resource availability requirements are met. However, resource continuity constraints cannot be represented directly in CPM networks, and so the uninterrupted utilization of resources from unit to unit cannot be assured. The schedule shown in Fig. 1 does provide for the continuous usage of the resource used by the *C* activities. Activity *C*1 begins on Day 3 and ends on Day 7, Activity *C*2 begins on Day 7 and ends on Day 10, and Activity *C*3 begins on Day 10 and ends on Day 14, so that the use of the resource is uninterrupted from Day 3 to Day 14. (Note that this continuous resource usage was neither required nor could have been anticipated).

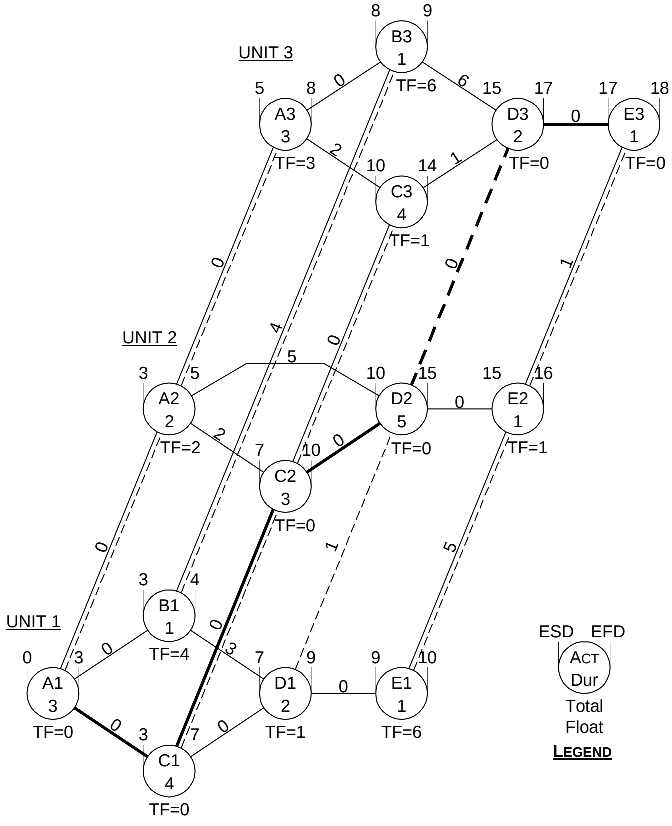


FIG. 1. CPM Network for Three Repetitive Units

For the *D* activities, the scheduled times do not provide continuous resource utilization. Activity *D*1 is scheduled to finish on Day 9, but the start of Activity *D*2 is not scheduled to begin until one day later (i.e., on Day 10). Therefore, there is a one day gap in the utilization of the resource needed for the *D* activities. Similarly, resource continuity is provided by the schedule of the *A* activities, but is not achieved for the *B* and *E* activities.

When uninterrupted utilization of resources is needed, activities having breaks in resource continuity can be rescheduled using their float times. For example, the one day Total Float for Activity *D*1 can be utilized, and *D*1 can be rescheduled to start on Day 8 and end on Day 10. In large projects with repeating activities, a complete activity-by-activity analysis and correction of the CPM network is required to ensure resource continuity, a process that is cumbersome and fraught with the possibility of error.

It may also be concluded from Fig. 1 that CPM networks for projects with repeating units of work have a ladder-like appearance, where each rung is a subnetwork that consists of the activities and precedence links for one unit. Because CPM diagrams show all of the linkages between similar activities in successive units, the number of links and nodes will likely be large and the network will appear unnecessarily complicated.

RSM SCHEDULE REPRESENTATION

In contrast to the complex CPM network for scheduling multiunit projects, an RSM schedule is presented graphically as an *X*-*Y* plot where one axis represents units, and the other time. The repetitive units may be assigned to either axis of the plot, the particular assignment being chosen for convenience and to clearly communicate the schedule information. For vertical construction projects, the repetitive units are usually discrete entities, such as houses, stores, apartments, or floors in high-rise construction, and work progress is measured in units completed. Hence, the units are typically shown along the *Y*-axis and time is shown along the *X*-axis. For horizontal construction projects, such as highways, pipelines, canals, tunnels, etc., work progress is measured in units of length and these units are shown along the *X*-axis to correlate with horizontal and vertical alignment charts, while time is shown along the *Y*-axis.

The repetitive units of the project must be arranged in some logical sequence along the chosen axis to define their pattern of repetition. This sequence may be accepted as a natural occurrence or may be established to suit some production need. For example, building floors must naturally be constructed one upon another, but houses in a development might be planned to follow in the order of their projected sale. Similarly, stations along a highway may follow in the natural numerical order from project start to project finish, or may be planned to recognize particular site or traffic conditions.

RSM ACTIVITY LOGIC

In addition to establishing the pattern by which repetitive units follow each other, it is necessary to identify the precedence constraints among the activities in each. To do so, a CPM precedence network is prepared for each typical repetitive unit, or if necessary, for each nontypical unit. These diagrams are similar to those shown for each unit in Fig. 1. The number of activities in the network for a repetitive unit is not particularly important and is determined by the nature of the unit. Resource considerations are temporarily ignored while preparing this diagram.

RESOURCE CONSIDERATIONS

Most activities require that several resources be employed together (e.g., a piece of equipment needs an operator). RSM assumes that only the most significant resource is associated with an activity, and that all activities have been defined using this assumption. It is also assumed that the same resource will be used for like activities in successive repeating units, and so each activity's resource must be consistent from unit to unit. For example, if an activity in the first unit requires a crew of carpenters, that activity in each succeeding unit will require the same crew of carpenters.

There are two important and often confused production rates associated with each activity: (1) a resource production rate; and (2) a unit production rate. The resource production rate for an Activity *A*, rpr_A, is the amount of work that can be accomplished by the resource in one time period. In equation form

$$rpr_A = \frac{Q_{Ai}}{T_{Ai}} \tag{1}$$

where rpr_A = resource production rate; Q_{Ai} = quantity of work in activity *A* in any repeating unit, *I*; and T_{Ai} = time needed to complete activity *A* in Unit *i*. Eq. (1) is most often used to estimate the activity duration, T_{Ai}, inasmuch as the quantity of work, Q_{Ai}, is taken from the plans and specifications and a standardized resource production rate, rpr_A, for the selected resource and method is taken from company databases or from any of several construction guides in common use in the construction industry.

The unit production rate is the number of repetitive units that can be accomplished by a resource during a unit of time. For an activity, *A*, in any repeating unit, *i*, the unit production rate, upr_{Ai}, can be expressed as

$$upr_{Ai} = \frac{1}{T_{Ai}} \tag{2}$$

where T_{Ai} = time needed to complete the unit. The unit production rate (and not the resource production rate) is the slope of a production line in an RSM diagram.

If (1) is solved for T_{Ai}, substituted into (2), and applied to any repeating unit, the following is obtained:

$$upr_{Ai} = \frac{rpr_A}{Q_{Ai}} \tag{3}$$

Observe that the unit production rate is directly proportional to the activity's resource production rate and inversely proportional to the quantity of work in the unit. For example, if rpr_A is expressed in square meters per day (m^2/d) and Q_{Ai} in square meters per floor (m^2/fl), then upr_{Ai} is in floors per day (fl/d). The resource production rate is an attribute of the resource and thus remains constant in any unit involving the same activity (i.e., the same crew will work at the same rate in every repeating unit regardless of the quantity of the work in the unit). Thus, upr_{Ai} may change from unit to unit as a function of the quantity of work, Q_{Ai}, though rpr_A does not.

Sometimes the quantity of work in activities that repeat from unit to unit is not the same in every unit (e.g., "Carpeting for Floor 2" may be twice as much as "Carpeting for Floor 1"). In such instances, the unit production rates will vary depending upon the amount of the work in each unit. For example, let Activities *C*1, *C*2, and *C*3 represent a case wherein the work quantity (e.g., the amount of carpet to be laid on each floor of a multistory project) in Unit 2 is twice that in Unit 1, and the quantity in Unit 3 is one-half that of Unit 1. In equation form:

$$Q_{C1} = \tfrac{1}{2}Q_{C2} = 2Q_{C3} \tag{4}$$

The unit production rates are then 1/2 unit per day (u/d) for activity *C*1, 1/4 u/d for *C*2, and 1 u/d for *C*3, or

$$upr_{C1} = 2upr_{C2} = \tfrac{1}{2}upr_{C3} \tag{5}$$

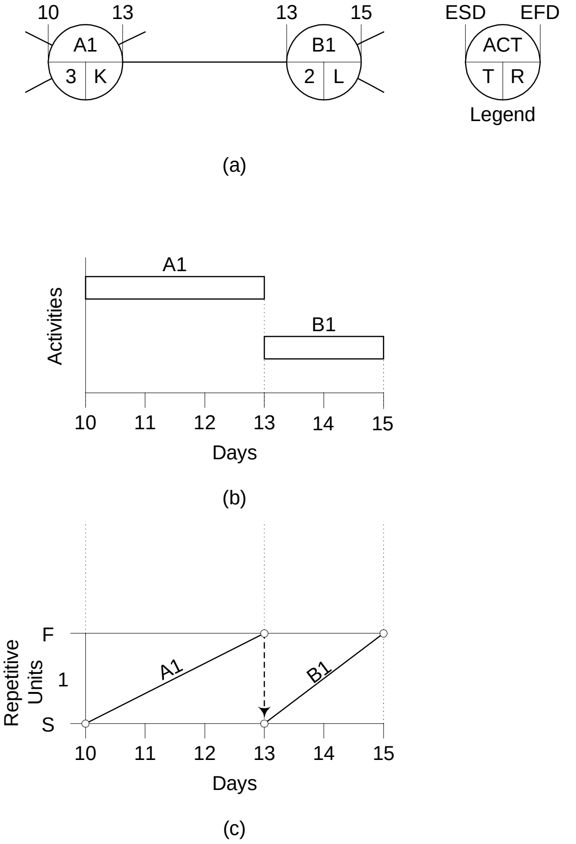
This means that the production line for the *C* activities consists of three linear segments, one for each unit, each having a different slope given by the corresponding upr_{Ci}.

CONVERGING PRODUCTION LINES IN RSM

Fig. 2(a) represents a pair of activities, *A*1 and *B*1, removed from a precedence network drawn for a project's Repetitive Unit 1, where the link relationship between the activities is finish-to-start (FTS). The time duration, *T*, the resource designation, *R*, the early start day, ESD, and the early finish day, EFD, are as shown in the legend. The values of *R* are expressed as alphabetic symbols to identify the particular resource being used by the activity.

These two activities are plotted as a bar chart in Fig. 2(b). They are plotted again in the form of an RSM diagram in Fig. 2(c). There is only one repetitive unit, and the zero point on the *Y* axis is designated by *S* to indicate the start of the unit. The finish of the unit is designated by *F*.

The inclined line drawn from the start of Activity *A*1 in Unit 1 to the finish of Activity *A*1 in Unit 1 represents the production line for Activity *A*1. In a similar manner, the production line for Activity *B*1 is drawn from its start at the end of Day 13 and the start of the unit to its finish at the end of Day 15 and the finish of the unit. The FTS precedence relationship between the activities is indicated by the dotted arrow at Day 13 drawn downward from



(a)

(b)

(c)

FIG. 2. Converging FTS Activities in RSM

the finish of Activity *A*1 to the start of Activity *B*1. Note that the unit production rate for Activity *A*1 is one-third unit per day (1/3 u/d), and for Activity *B*1 the rate is 1/2 u/d. These rates will be recognized as the mathematical slopes of the respective production lines.

The same pair of activities extended over three repetitive units is plotted in the form of a bar chart in Fig. 3(a). Each unit contains the two activities, and the numeral associated with each activity identifies the unit in which it is scheduled. Because only technical precedence logic is employed, there is a one day lag between the *B* activities from one unit to the next.

Fig. 3(b) shows a unit-by-unit RSM plot of the same activities with the FTS relationships shown by the downward-pointing arrows. The production lines plotted for Activities *A*1–*A*3 form a continuous straight line beginning at the end of Day 10 and ending on Day 19. Because each *A* activity uses Resource *K* and has a unit production rate of 1/3 u/d, it follows that the production line for the three *A* activities also has the same unit production rate of 1/3 u/d. While no attempt was made to provide for the continuous utilization of the resource from unit to unit, the continuous production line for the *A* activities ensures that this is true.

The production lines for Activities *B*1–*B*3 do not form a continuous production line when plotted unit by unit because of the lags between the *B* activities. To make a continuous production line for the *B* activities and provide for the uninterrupted utilization of resources, the start of Activity *B*1 must be delayed by two days and the start of Activity *B*2 must be delayed by one day. The resulting production line for the *B* activities is shown in Fig. 3(b) as a dashed line beginning at the end of Day 15 and continuing through Unit 2 then extending as a solid line through Unit 3 to finish on Day 21. Because the unit production rate for each unit's *B* activity is 1/2 u/d, the unit production rate of the production line for the *B* activities is also 1/2 u/d.

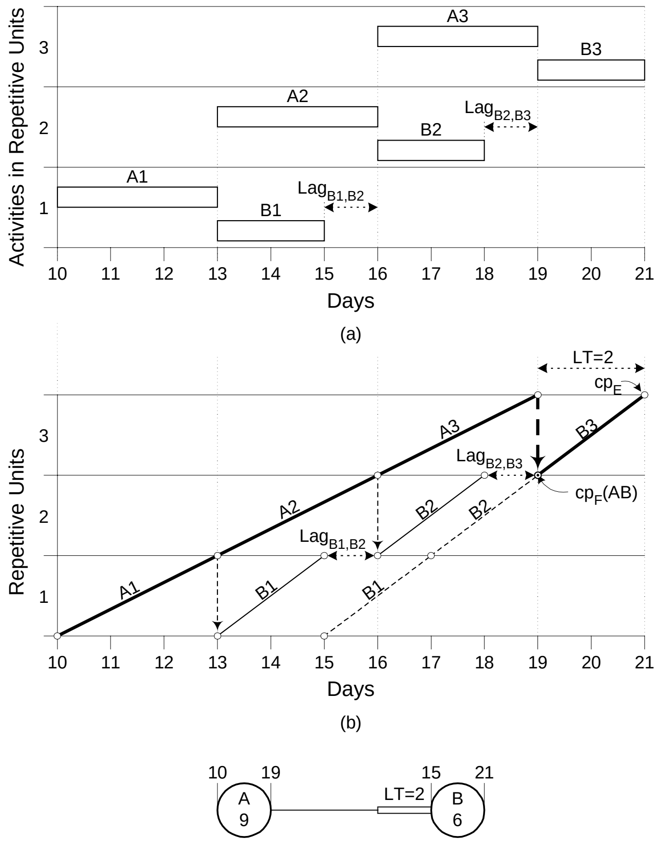


FIG. 3. Bar Chart and RSM Diagram for Three Units With Converging FTS Activities

Notice that the two continuous production lines converge toward the finish of Unit 3 because the unit production rate of the *B* line is greater than that of the *A* line. Also note that at Day 19 and the beginning of Unit 3, the end of the FTS arrow between the finish of Activity *A*3 and the start of Activity *B*3 controls the start of Activity *B*3, and subsequently, the position of the *B* line. This location, or control point, has been labeled $cp_F(AB)$ where the subscript *F* stands for finish and signifies the last unit in the sequence, and the letters *A* and *B* show the dependency of Activity *B* upon Activity *A*. This illustrates a basic RSM principle:

> When the unit production rate of an activity's production line is greater than the unit production rate of the preceding activity's production line, the two production lines will tend to converge as the number of units increases. Owing to the desired continuous utilization of resources from unit to unit, this convergence tends to place any dependency control between the activities toward the last unit in the sequence.

With the preceding principle in mind, a simple procedure for constructing the production line for the *B* activities suggests itself. First establish the control point $cp_F(AB)$ at the start of Activity *B*3 and then draw the continuous production line for *B* through it.

Because Activity *B*3 is the last activity in the sequence, another control point, called cp_E, at the end of Activity *B*3 can also serve as a point through which the *B* production line may be drawn. (The subscript *E* in cp_E stands for the end of the activity and the production line.)

The two days shown in Fig. 3(b) between the end of Activity *A*3 and the end of Activity *B*3 at cp_E is a lead time (LT), that relates the finish of the *B* production line to the finish of the *A* production line. This corresponds to a finish-to-finish (FTF) relationship shown in the equivalent CPM overlapping diagram of Fig. 3(c) where the lead time for the link between the *A* and *B* activities represents the amount of time that must remain in the *B* activity after the finish of the *A* activity. In this context, the two-day duration of Activity *B*3 represents the amount of time that must remain in *B* after the finish of Activity *A*3 and sets the lead time at 2 days. Thus, the control point, cp_E, can be positioned two days after the finish of Activity *A*3, and the *B* production line can be drawn through it.

DIVERGING PRODUCTION LINES IN RSM

Fig. 4(a) is similar to Fig. 3(a). It is a bar chart of another pair of activities removed from a precedence network for a project's Repetitive Unit 1. These two activities are extended over three repetitive units with the activities grouped by unit. Each *A* activity has a duration of 2 days and each *B* activity has a duration of 3 days. The precedence relationship between the activities in each unit is FTS, and each activity is shown in its scheduled early start position when only technical precedence logic is used. Thus, there is a lag of one day between Activities *A*2 and *B*2, and a lag of two days between Activities *A*3 and *B*3.

Fig. 4b is an RSM unit-by-unit plot of the same activities, with the FTS relationships indicated by the downward-pointing dotted arrows at Days 12, 14, and 16. The lags shown between the finish of each *A* activity and the start of its related *B* activity are the same as those shown in Fig. 4(a). As plotted in Fig. 4(b), the production lines for both *A* and *B* are continuous and ensure the uninterrupted utilization of resources even though no deliberate attempt was made to achieve resource continuity.

Also note that in Fig. 4(b), the unit production rate of the *B* production line, 1/3 u/d, is smaller than the unit production rate of the *A* production line, 1/2 u/d. The two unit production lines

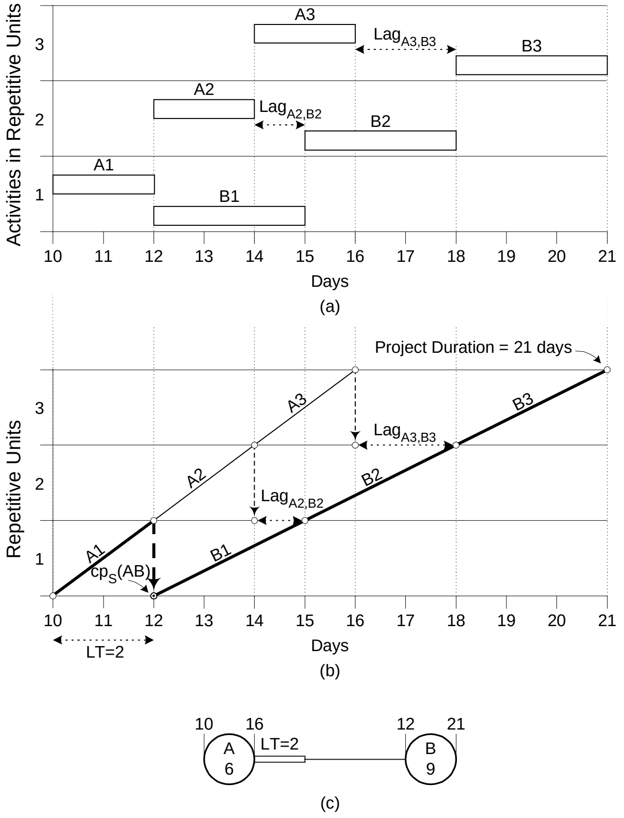


FIG. 4. Bar Chart and RSM Diagram for Three Units With Diverging FTS Activities

therefore diverge and the FTS control between the two is found at Day 12 in Unit 1. This control point is labeled $cp_S(AB)$, where the subscript *S* stands for start and signifies the first unit in the sequence, and the letters *A* and *B* show the dependency of Activity *B* upon Activity *A*. This illustrates another basic RSM principle:

> When the production rate of an activity's production line is smaller than the production rate of the preceding activity's production line, the two production lines will tend to diverge as the number of units increases. Owing to the desired continuous utilization of resources from unit to unit, this divergence tends to place any dependency control between the activities toward the first unit in the sequence.

The two days shown in Fig. 4 between the start of Activity *A*1 and the start of Activity *B*1 at $cp_S(AB)$ is a LT that relates the start of the *B* production line to the start of the *A* production line. This corresponds to a start-to-start relationship shown in the equivalent CPM overlapping diagram of Fig. 4(c) where the lead time for the link between the *A* and *B* activities represents the time to accomplish the work required in the *A* activity before the *B* activity can begin. In this context, the two day duration of Activity *A*1 represents the amount of time that must elapse before the start of Activity *B*1 and sets the lead time at 2 days. Thus, $cp_S(AB)$ can be positioned two days after the start of Activity *A*1. The *B* line passes through $cp_S(AB)$ and ends at the finish of Activity *B*3 on Day 21 and sets the duration of the project at 21 days.

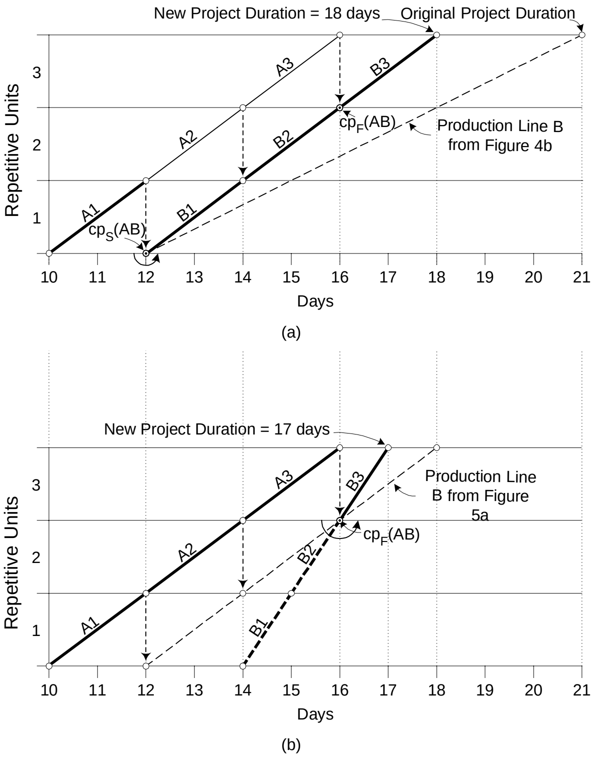


FIG. 5. Effects of Increasing Unit Production Rates in RSM DiagramsWith FTS Activities

EFFECTS FROM CHANGING UNIT PRODUCTION RATES

Suppose that the crew for each *B* activity of Fig. 4 is increased by 50%. This change reduces each *B* activity duration to two days and increases each unit production rate to 1/2 u/d, the same as that of each *A* activity. An RSM diagram for the three units of Fig. 4(b) with this revised unit production rate is shown in Fig. 5(a) along with the dashed production line from Fig. 4(b).

The control point, $cp_S(AB)$, still controls the position of the *B* production line that now lies parallel to the line for the *A* activities. Therefore, increasing the unit production rate of the *B* production line from 1/3 to 1/2 u/d is tantamount to rotating the production line about this control point. A curved arrow at $cp_S(AB)$ signifies this rotation. The project duration is reduced from 21 days to 18 days, and the FTS arrow at the beginning of Activity *B*3 defines another control point, $cp_F(AB)$, through which the new *B* production line passes.

If the resources of each *B* activity are doubled over those shown in Fig. 5(a), the unit production rate of the *B* activities becomes 1 u/d and causes the *A* and *B* lines to converge. A further rotation of the *B* production line about $cp_S(AB)$ would violate the FTS relationships at Days 14 and 16, and so the control of the *B* line must shift to $cp_F(AB)$ at the beginning of Unit 3. Fig. 5(b) shows this shift in control point and the rotation of the *B* production line about $cp_F(AB)$ where the curved arrow refers to the rotation of the line. The *B* line now begins at the end of Day 14 and sets the project duration at 17 days, one day shorter than in Fig. 5(a).

RSM DIAGRAM CONSTRUCTION

The previous examples use only two production lines so that basic RSM principles may be illustrated clearly. However, they are too small to demonstrate the construction of an RSM diagram. A project with six repeating units, each having six discrete activities, has been chosen for this purpose.

The CPM precedence diagram for the activities in the first unit is shown in Fig. 6. The solution for the early start and finish days, the critical path, and the 12-day duration is shown on the network. Of this information, only activity durations and

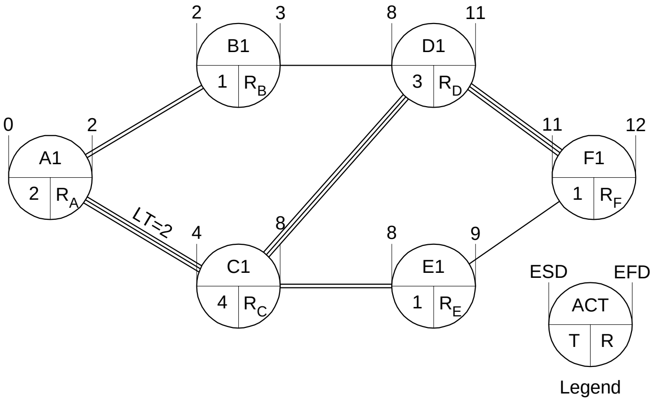


FIG. 6. Precedence Diagram for Unit One of a Six Unit Project

precedence relationships are required for the construction of an RSM diagram. Similar diagrams, not shown here, establish these same requirements for each of the remaining units.

Fig. 7 is the RSM diagram for all the activities in the project. Because the first activity in the precedence diagram is *A*, the first production line plotted in Fig. 7 is that for the *A* activities. The unit production rate for the *A* production line in Units 1 and 2 is 1/2 u/d. However, the amount of work to be done in Units 3 and 4 by the *A* activities is twice the work to be done in Unit 1. Hence, under the assumption of constant resource production rates, the unit production rate of the *A* production line in Units 3 and 4 will be 1/4 u/d. The amount of work in Units 5 and 6 is the same as in Unit 1 and the unit production rate for the *A* line is again 1/2 u/d. The *A* production line that begins at time zero and ends at Day 16 consists of three connected linear segments with different slopes. Even so, the continuity of these segments ensures the continuous utilization of the resource needed by the *A* activities.

In the precedence diagram of Fig. 6, the *B* and *C* activities are not related, but each is a successor to the *A* activity. The next choice for plotting a production line is therefore arbitrary, and the line for the *B* activities is selected. The FTS relationships between the *A* and *B* activities must prevail at every unit, although there may be link lags between these activities at any unit.

The unit production rate for each of the *B* activities is 1 u/d, which is greater than either of the unit production rates for the *A* activities, and so the production lines converge. The *B* production line is therefore controlled at the start of Unit 6, where the control point, $cp_2(AB)$, is shown at the dotted FTS arrow on Day 16. To maintain the continuity of resources for the *B* activities, the production line for the *B* activities must pass through this point.

It sometimes happens that an interruption in resource continuity may need to be planned to meet some known or predicted circumstance. In this instance, the *B* activities are performed by a subcontractor from a different area, and on each trip to the site, the subcontractor's truck can deliver materials sufficient for completing only three units. The production line for the *B* activities is therefore interrupted between Unit 3 and Unit 4 to accommodate the delivery, and the *B* line has two segments.

Control point $cp_2(AB)$ can continue to control the position of the upper segment of the *B* production line, but another control point, $cp_1(AB)$, controls the leftmost possible position of the lower segment. This point is located at the FTS arrow on Day 8 in Unit 3, and the production line for the *B* activities in the first three units has been drawn through it to maximize the break time available to the subcontractor. The production line for the first three *B* activities is now planned to start at the end of Day 6. The dotted line labeled "Work Break" between Days 9 and 14 illustrates the planned interruption.

The production line for the *C* activities is plotted next. The unit production rate for all *C* activities is 1/4 u/d which is smaller than, or equal to, that of the *A* activities. Hence, the production lines for the *A* and *C* activities diverge and the plot of the production line for the *C* activities is controlled at Unit 1. The FTS relationship between Activity *A*1 and Activity *C*1 is shown by the downward dotted arrow at Day 2. There is a lead time shown on the link between the *A* and *C* activities in Fig. 6, and this lead time is shown between Days 2 and 4 in Fig. 7. Lead times must prevail in each repeating unit, but because the production lines diverge, only the lead time in the first unit needs to be recognized here. Consequently, the control point, $cp(AC)$, is found at the end of Day 4 in Unit 1 and marks the beginning of the production line for the *C* activities.

This project does not have an Activity *C* in Unit 5, so when Activity *C* in Unit 4 is completed, Activity *C* in Unit 6 can begin, and the *C* production line shifts upward as indicated by the dotted line in Unit 5 at Day 20. This displacement interrupts the *C* production line at Unit 5, but does not create an interruption in the

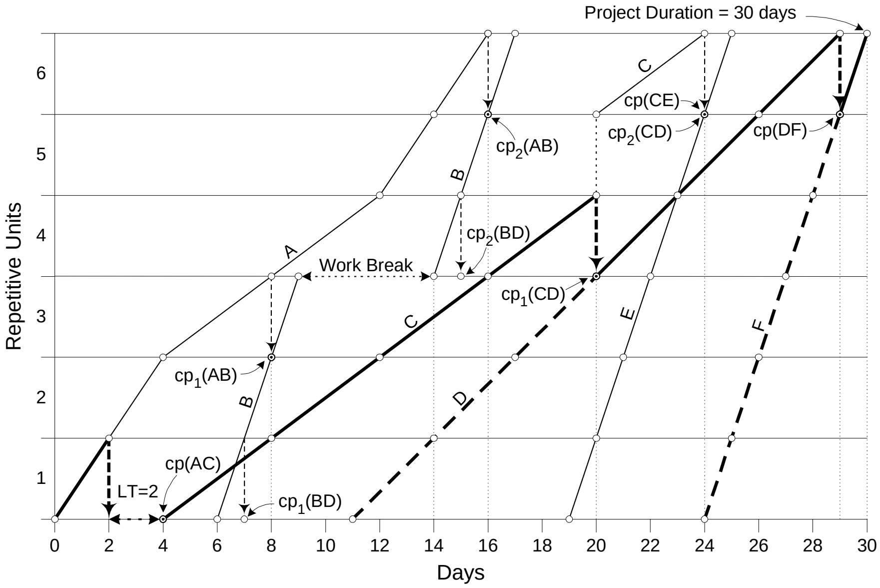


FIG. 7. RSM Diagram for a Six Unit Project

continuous utilization of its resource. It may be observed that the production lines for the *B* and *C* activities appear to cross each other. This has no significance because the two activities are not related to each other.

The next production line chosen for plotting is for the *D* activities. It is selected because Activities *B* and *C* are the predecessors to Activity *D* in Fig. 6, and because these two production lines have already been drawn and at least one of them will control the position of the *D* line.

All possible control points from each predecessor production line must be considered when establishing a control point to position a production line. First, consider the relationships between the *B* and *D* production lines. The unit production rate for the *B* line is 1 u/d and for the *D* line is 1/3 u/d, so these lines diverge. One possible control point, labeled $cp_1(BD)$, might be located at Day 7 and the start of Unit 1. Because of the work break, another possible location for a control point might be at $cp_2(BD)$ at Day 15 and Unit 4. Now consider the relationship between the *C* and *D* production lines. The unit production rate for the *C* line is 1/4 u/d and for *D* line it is 1/3 u/d, so the production lines converge. Because of the skip in units on Day 20 and the discontinuity in the *C* line, there are two possible control points; one located at Day 20 and the start of Unit 4, and the other at Day 24 and the start of Unit 6.

If the control point at Day 7 is used to control the position of the *D* line, none of the FTS relationships between Activities *B* and *D* would be violated, but all of the relationships between Activities *C* and *D* would be. Further, if the control point at Day 15 is used, the FTS relationship between Activities *B* and *D* would be violated in Unit 1 in addition to all those between Activities *C* and *D.* If the control point at Day 24 is used, the FTS relationships between all of the *B* and *D* activities can be met, but the relationships between *C* and *D* cannot be met at Units 3 and 4. The remaining possibility is to use control point $cp_1(\mathrm{C}D)$ at Day 20 and the start of Unit 4. This point allows all FTS relationships to be satisfied, and the production line for the *D* activities is drawn through it.

The production line for the *E* activities is considered next. Its unit production rate is 1 u/d, and it is related only to the production line for the *C* activities. As before, two control points are possible, one at Unit 4 and the other at Unit 6. In this instance, control point $cp(CE)$ is found in Unit 6 at Day 24, and the *E* production line is drawn through it. The fact that the production lines for the *E* and *D* activities intersect is of no consequence because they are not related to each other.

The production line for the *F* activities is the last to be drawn. It depends upon the lines for activities *D* and *E.* The unit production rate for the *F* line is 1 u/d, which is larger than, or equal to, either of the rates for these predecessors. This convergence forces the control point to be in Unit 6. Because the production line for the *D* activities finishes later than that of the *E* activities in Unit 6, control point $cp(DF)$ is clearly established at Day 29 in Unit 6. The finish of the production line for the *F* activities marks the end of the project and sets the project duration at 30 days.

CONTROLLING SEQUENCE

In CPM networks, a critical activity is defined as one that, if delayed, will delay the project, and a chain of these critical activities extending from project start to project finish is called the critical path. Adding the durations of the critical activities along this path establishes the minimum project duration consistent with the technical precedence and resource availability constraints explicitly expressed in the network. However, the determination of the project duration from a critical path does not apply in RSM because of the additional resource continuity requirement. This requirement forces noncritical activities to become critical, and may cause noncritical activities to be included in the chain of activities that controls project duration.

In RSM, the chain, or sequence of activities, that establishes the minimum project duration is called the "controlling sequence." This sequence maintains all technical precedence, resource availability, and resource continuity constraints, and passes through control points that switch the sequence from production line to production line. Some of the activities on the controlling sequence may be critical in the CPM sense, and some may not. If the activity is critical, a delay in the completion of the activity delays the completion of the project. If the activity is noncritical, a delay in the completion of the activity does not delay the completion of the project, but introduces discontinuities in resource utilization.

The controlling sequence through the six-unit project of Fig. 7 is found by tracing along production lines from the project finish to the project start while shifting from one production line to the next at the defined control points. The trace begins at the finish of the *F* production line at Day 30. It moves downward along the *F* line to control point $cp(DF)$. The trace then shifts to the production line for the *D* activities in Unit 6 and moves downward again to control point $cp_1(CD)$. It shifts again to the *C* line and moves down to control point $cp(AC)$ at Day 4. At this point it moves through the lead time and the FTS arrow to the *A* production line in Unit 1. The trace finishes at Day zero, which is the start of the *A* line.

The individual activities that comprise this controlling sequence are Activity *A* in Unit 1; Activities *C* in Units 1, 2, 3, and 4; Activities *D* in Units 4, 5, and 6; and Activity *F* in Unit 6. These activities also happen to be critical (i.e., none can be delayed without delaying the project), and they form a critical path that is the same, in this instance, as the controlling sequence. These activities are shown in Fig. 7 by a heavy solid line.

Other activities are critical only because they are scheduled to provide resource continuity. For example, Activities *D*1, *D*2, and *D*3 are each scheduled to start at Days 11, 14, and 17, respectively, to ensure the continuous usage of their resource. If the completion of any one of them is delayed, then part of the controlling sequence and the completion of the project will be delayed. Activities *D*1, *D*2, and *D*3 are therefore critical by definition only because of their RSM schedule. Similarly, Activities *F*1–*F*5 are critical because they are scheduled to provide continuous usage of their resource. Activities such as these are called "resource critical activities" and they are shown in the figure with heavy dashed lines.

Activities *A*2–*A*6, *B*1–*B*6, *C*6, and *E*1–*E*6 are not critical. A delay in any one of these will not delay the project, but may cause an interruption of resources from unit to unit.

REDUCING PROJECT DURATION

The creation of a project schedule deemed satisfactory for construction is an iterative process. The first plan probably is not satisfactory for any number of reasons, and several adjustments to logic, resource usage, resource quantities, etc., will need to be made. One principal reason to adjust the plan is to reduce the project duration.

Minimum project durations can theoretically be achieved by adding resources to some activities and subtracting resources from others until all unit production lines have the same unit production rate and are parallel to each other. This ideal minimum, however, cannot always be achieved because construction resources can only be expressed in integer form. For example, there cannot be 3.7 workers on a crew; there can be either three or four. Similarly, there cannot be 1.5 pavers on a highway project; there can be either one or two. Moreover, a

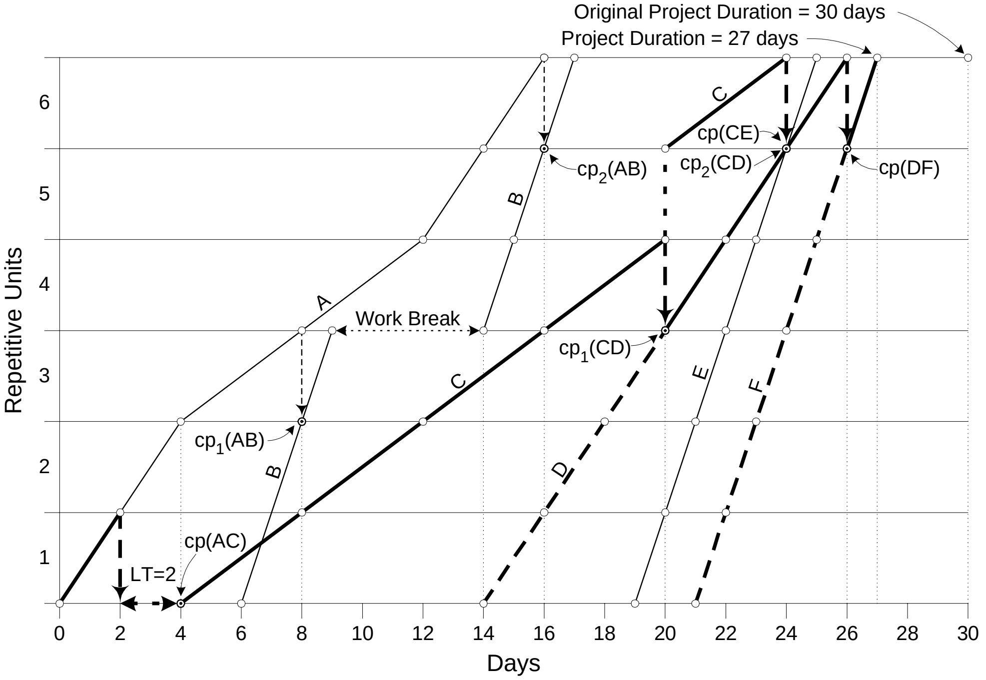


FIG. 8. RSM Diagram for a Six Unit Project With Unit Production Rate of D Line Increased

production line may be composed of several linear segments that have different slopes because of different quantities of work in each of the repetitive units. Consequently, unit production lines cannot always be parallel, and a more pragmatic approach needs to be employed.

The procedure for reducing the project duration is best illustrated by example. Assume that the duration of 30 days for the project in Fig. 7 is not acceptable, and that a review of the schedule is needed to seek a reduction in time. An examination of the figure shows that the *C* and *D* activities are major contributors to the 30-day project duration, so that it appears that the duration of the project can be reduced if the unit production rate of one of these two production lines is increased.

Suppose further that it is possible to increase the unit production rate of the *D* production line to 1/2 u/d as shown in the RSM diagram of Fig. 8. The *A*, *B*, and *C* production lines are unchanged. The new rate for the *D* line is greater than the rate for the *C* line, so the two lines converge as before, and the previously used control point, $cp_1(CD)$ at Day 20, is still the control point for the *D* line. The line is rotated about this point and the new production line for the *D* activities causes the *D* line to pass through both control points $cp_1(CD)$ and $cp_2(CD)$. Any further increase in unit production rate for the *D* line will cause a rotation about $cp_2(CD)$.

The *E* production line remains in its position because its control point, *cp*(*CE*), is unchanged. However, the *F* production line is affected by the rotation of the *D* line. The new unit production rate of the *D* line is still less than the rate of the *F* line, so there is still convergence between the two lines, but the control point *cp*(*DF*) has been shifted back to Day 26. The *F* production line is drawn through this control point and sets the project duration at 27 days, or three days less than the original plan.

The controlling sequence in Fig. 8 is found by tracing backward from the end of the project as was done earlier. The trace begins at Day 27 with the finish of the *F* production line and moves downward to *cp*(*DF*), where it shifts to the *D* line and moves downward until $cp_2(CD)$ is reached. The trace splits at this point and one branch shifts to the *C* line while the other branch continues down the *D* line to control point $cp_1(CD)$, where it shifts to the *C* line. The trace for both branches continues down the *C* line to $cp_1(AC)$ and then shifts to the *A* line. Note that this creates two controlling sequences, one passing through Activity *C*6 and the other through Activities *D*4 and *D*5. All of the activities on these controlling sequences are critical and also form two critical paths. As before, these controlling sequences and critical paths are shown in the figure by a heavy solid line. Activities *D*1–*D*3 and *F*1–*F*5 are also critical but do not belong to either the controlling sequence or to the critical path, and so they are shown by a heavy dashed line.

PARADOX

As another alternative, suppose the unit production rate of the *C* line is increased instead of that of the *D* line because the unit production rate of the *C* line is the smallest rate of the two. Fig. 9 shows the RSM diagram with the rate of the *C* line increased from 1/4 to 1/2 u/d. This new rate is greater than, or equal to, either of the rates of the *A* production line, and the lines will converge. This change from divergence to convergence causes a shift of the control point relating the two activities. The *A* and *B* production lines remain the same as before, but the new control point, *cp*(*AC*) shown in Fig. 9, is located at Day 18 in Unit 6.

The unit production rates for the *D*, *E*, and *F* lines also remain as in Fig. 7, but because of the change in the rate for the *C* line, the *C* and *D* production lines diverge and the control point, *cp*(*CD*), relating lines *C* and *D* shifts to Day 12 in Unit 1. The production line for the *D* activities now begins at the end of Day 12 and ends on Day 30, which causes the project duration to be increased by one day, to 31 days. Thus, increasing the unit production rate of the *C* activities does not shorten the project duration as expected, but actually increases it. This paradox is explained next.

As before, the controlling sequence of activities in Fig. 9 is found by tracing from the end of the project back down along the *F* production line until control point *cp*(*DF*) at Day 30 is reached. The trace then shifts to the *D* line and follows downward to control point *cp*(*CD*) at Day 12. At this point, the trace moves up the FTS arrow to the *C* line that is controlled by *cp*(*AC*) at Day 18 and Unit 6. It is important to note that control point *cp*(*AC*) occurs later than *cp*(*CD*). As a result, the trace of the controlling

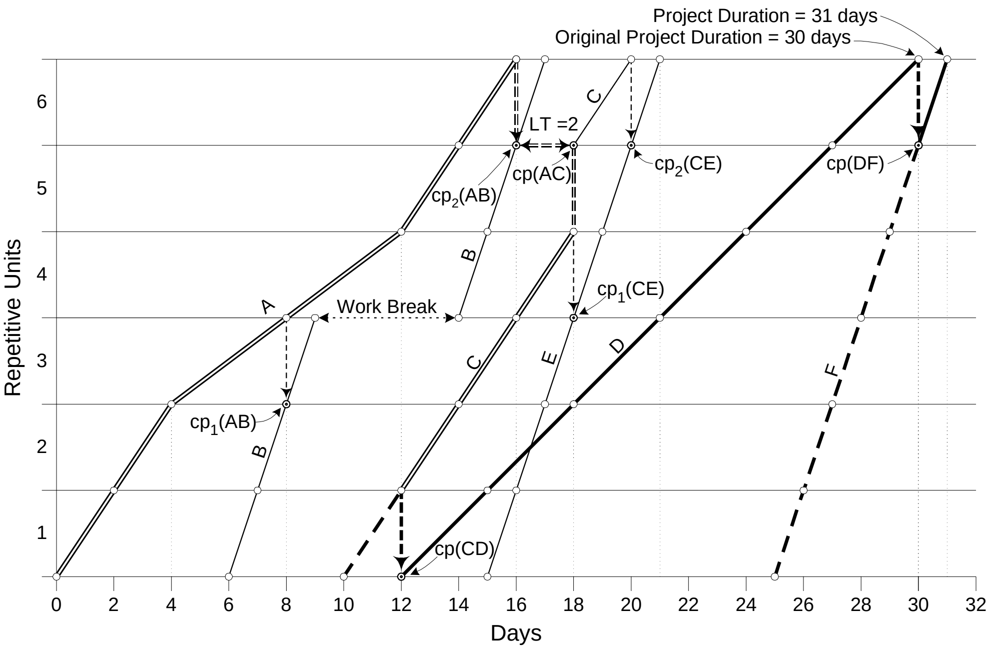


FIG. 9. RSM Diagram for a Six Unit Project With Unit Production Rate of C Line Increased

sequence traverses the *C* line forward in time rather than backward. The trace next follows the lead time between Activities *A* and *C* and upward on the FTS arrow to the *A* line. The trace follows down the *A* line to the start of the project. The controlling sequence consists of the activities represented by the double and heavy continuous lines.

All of the *A* activities and the *C* activities from Units 2–4 are shown with double lines in Fig. 9. This notation indicates that these activities are part of the controlling sequence, but are not critical because they have floats that can be utilized if any are delayed. All of the *D* activities and the *F* activity in Unit 6 are shown with heavy solid lines as before to illustrate that they are part of the controlling sequence and are also critical. Activity *C* in Unit 1 and Activities *F* in Units 1–5 are shown with heavy dashed lines to indicate that they are critical, but do not belong to the controlling sequence. The remaining activities in the project are shown with plain solid lines because they are neither critical nor lie on the controlling sequence.

The paradox where expediting an activity results in an increase in project duration, can be explained by reference to Fig. 9. When the unit production rate of the *C* line was increased, the line rotated counterclockwise and the new upr_C became equal to or greater than upr_A. Hence, control point *cp*(*AC*) is now at Unit 6. Also, upr_C became greater than upr_D and control point *cp*(*CD*) now appears at Unit 1 and Day 12. This change in the location of *cp*(*CD*) forced the position of the start of the *D* line to be one day later than before. In turn, this forward shift in the *D* line caused the project duration to be increased by one day. To draw on a physical analogy, it's as though the *C* line kicked the foot of the *D* line downstream.

CONCLUDING REMARKS

Construction contractors often need to model multiunit projects wherein activities repeat from unit to unit. These activities need to be scheduled so that their required resources are continuously used once they arrive on the construction site. The use of typical CPM scheduling techniques cannot ensure this continuity in resource utilization because only technical precedence and resource availability constraints are shown in CPM networks. RSM recognizes the additional resource continuity constraint that cannot be shown in a network, and thus provides for continuous resource usage.

RSM is presented here as a scheduling method that simplifies and generalizes several variously named, multiunit scheduling techniques that have been cited in past publications. It incorporates commonly accepted activity precedence concepts from CPM, and can be applied to both vertical and horizontal projects that may contain either discrete or continuous activities.

An RSM schedule is presented graphically as an *X*-*Y* plot of unit production lines that continue across designated units of the project. One axis of the plot represents units, and the other represents time, and the repetitive units may be assigned to either axis, the particular assignment being chosen for convenience and to clearly communicate the schedule information. Typically, a resource production line appears in the diagram as a continuous straight line. However, some activity segments of the line may have different slopes if the work content in repeating units is not uniform.

The construction of RSM schedules involves the positioning of successive unit production lines by using the new concept of control points. As shown earlier, there is a specific point along each production line that controls the schedule position of its successor production line. This point, called a control point, tends to be located toward the first unit in the sequence of units if the lines diverge, and toward the last unit in the sequence if the lines converge. These control points have significance in the determination of the project duration, and serve as points of rotation for unit production lines whose resource rates are increased or decreased.

RSM also introduces a new concept for the determination of the project duration. As with all projects, the duration must be determined by some sequence of activities that extends from project start to project finish. This sequence in RSM is called the controlling sequence and includes the activities of the first production line from project start until the first control point is reached. It then switches to the next production line and includes all activities on that line until the next control point is found. The sequence continues to include activities in this fashion, switching from production line to production line at control points, until reaching the end of the project. An RSM controlling sequence may include both critical and noncritical activities. Conversely, activities can be critical because of resource continuity (resource critical), and, thus, not be part of the controlling sequence.

The unit production rate of any activity can be increased or decreased by altering the composition of the crews or equipment needed to carry out the activity. This causes the associated unit production line to rotate about a control point and to increase or decrease the project duration. However, care must be taken in choosing the activity and resource to change; a poor choice may shift the location of the controlling point for the production line and result in an unexpected project length. Shortening the duration of an activity may end up increasing the duration of the project.

RSM is a practical scheduling methodology. It uses customary work methods and crews to define repetitive activities that can be arranged in any desired pattern. RSM diagrams are easy to prepare and understand, and the unique concepts of control points and controlling sequence are quickly comprehended. The project duration, along with the start time, the finish time, and the critical status of each activity are quickly found from the diagram. Thus, RSM has all the necessary performance characteristics to serve as a convenient and practical tool for scheduling multiunit projects.

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APPENDIX II. NOTATION

The following symbols are used in this paper:

A, B, C = activity designation or production line designation;
$cp(AB)$ = designation for control point between production lines A and B;
F = finish of a repetitive unit;
i = unit designation;
K, L = resource designation;
Q = quantity of work in a repetitive unit;
rpr = resource production rate;
S = start of a repetitive unit;
T = time needed to complete one unit;
u/d = units per day; and
upr = unit production rate.